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— EDUCATION —

Specialist Mathematics Examination 2

Solutions Book

VCE Trial Examination – Free

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Section A – Multiple-choice questions

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Question 1 (D)

The converse of a conditional statement is obtained by switching the hypothesis and the conclusion. That is: ‘If an animal does not eat meat, then it is a herbivore.’

Question 2 (C)

Since $x, y \in \left(0, \frac{\pi}{2}\right)$, we have

$$\cos(x) = \sqrt{1-\sin^2(x)} = \frac{5}{13} \quad \text{and} \quad \sin(y) = \sqrt{1-\cos^2(y)} = \frac{3}{5}.$$

Hence, using the compound angle identity for sine,

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y) = \frac{12}{13}\cdot\frac{4}{5} - \frac{5}{13}\cdot\frac{3}{5} = \frac{33}{65}.$$

Question 3 (A)

Using the definition of the absolute value function, the rule of f is given by

$$f(x) = \begin{cases} -a+b, & x \geq b \\ 2x-a-b, & a \leq x < b \\ a-b, & x < a \end{cases}.$$

Since $a \leq b$, it follows that $\operatorname{ran}(f) = [a-b, -a+b]$.

Question 4 (C)

The function `inMandelbrot` calculates elements of a sequence z_n generated by the recursion relation $z_{n+1} = z_n^2 + c$, where $z_0 = 0$. The function returns `False` if $|z_n| > 2$ for any index $n \leq$ `max`.

`n`	0	1	2	3	4	5
`z`	0	$\frac{1}{2}+\frac{1}{2}i$	$\frac{1}{2}+i$	$-\frac{1}{4}+\frac{3}{2}i$	$-\frac{27}{16}-\frac{1}{4}i$	$\frac{841}{256}+\frac{43}{32}i$
`abs(z)`	0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{5}}{2}$	$\frac{\sqrt{37}}{4}$	$\frac{\sqrt{745}}{16}$	$\frac{\sqrt{825617}}{256}$
`abs(z)>2`	No	No	No	No	No	Yes

Hence, the smallest `max` which returns `False` is 5.

Question 5 (C)

Observe that $w^{n!} = 1$ for all $n \geq 3$ since $n!$ is divisible by 6 for such n. Therefore,

$$w^{1!} + w^{2!} + w^{3!} + \cdots + w^{2025!} = w + w^2 + 2023 = 2023 + \sqrt{3}i.$$

Question 6 (D)

By the conjugate root theorem, $(z-2-i)^2$ and $z-3i$ are also factors of $p(z)$, so its minimum degree is 6.

Question 7 (B)

Let V cm^3 be the volume of the balloon after t seconds. Then, we have

$$V = \frac{4}{3}\pi r^3 \implies \frac{dV}{dr} = 4\pi r^2,$$

and so by the chain rule,

$$\frac{dr}{dt} = \frac{dr}{dV}\frac{dV}{dt} = \frac{1}{4\pi r^2} \cdot 2 = \frac{1}{2\pi r^2}.$$

Question 8 (B)

Using integration by parts, we get

$$I_n = \left[-x^n e^{-x}\right]_0^1 - \int_0^1 -nx^{n-1}e^{-x} = -e^{-1} + nI_{n-1}.$$

Question 9 (A)

The graph of f has a point of undulation at $x = 0$, so f'' should vanish at 0, but not change sign. The graph of f has a point of inflection at some point $x = a > 0$. The function f is concave on $(-\infty, a]$ and and convex on $[a, \infty)$, so the sign of f'' should change from negative to positive at a.

Question 10 (C)

Using Euler's method with a step size of 0.1, we get

$$\begin{aligned} y(0.1) &\approx y_1 = 1 + 0.1\left(0^2 - \sqrt{1}\right) = 0.9 \\ \implies y(0.2) &\approx y_2 = 0.9 + 0.1\left(0.1^2 - \sqrt{0.9}\right) = 0.8061 \quad \text{(4DP)}. \end{aligned}$$

Question 11 (A)

The volume of water in the tank after t minutes is given by $V(t) = 50 + (10-5)t$, so by the conservation of mass,

$$\frac{dx}{dt} = 0.1 \cdot 10 - \frac{x}{V(t)} \cdot 5 \implies \frac{dx}{dt} + \frac{x}{10+t} = 1.$$

Question 12 (C)

The acceleration of the particle is given by

$$a(x) = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(18 - \frac{1}{2}x^2\right) = -x \implies a(-2) = 2 \text{ m s}^{-2}.$$

Question 13 (A)

Since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are not proportional, the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent if and only if there exist $\lambda, \mu \in \mathbb{R}$ such that

$$\underset{\sim}{c} = \lambda \underset{\sim}{a} + \mu \underset{\sim}{b} \iff \begin{cases} -\lambda + 2\mu = m \\ 2\lambda - \mu = n \\ \lambda - \mu = 0 \end{cases} \iff m = n = \lambda = \mu.$$

Question 14 (A)

For some $\alpha, \beta, \gamma \in (0,1)$, we have

$$\overrightarrow{OP} = \alpha\overrightarrow{OQ} = \overrightarrow{OB} + \beta\overrightarrow{BM} = \overrightarrow{OA} + \gamma\overrightarrow{AN}.$$

Focusing on the equation involving α and β, we get

$$\alpha\left(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}\right) = \underset{\sim}{b} + \beta\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right) \implies \frac{1}{2}(\alpha - \beta)\underset{\sim}{a} + \frac{1}{2}(\alpha + 2\beta - 2)\underset{\sim}{b} = \underset{\sim}{0}.$$

Since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are linearly independent, it follows that

$$\frac{1}{2}(\alpha - \beta) = \frac{1}{2}(\alpha + 2\beta - 2) = 0 \implies \alpha = \beta = \frac{2}{3} \implies \overrightarrow{OP} = \frac{1}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}.$$

Question 15 (C)

The points $P(4,0,0)$ and $Q(-2,0,0)$ lie on the respective planes. The distance between the planes is the magnitude of the scalar resolute of $\overrightarrow{PQ}$ in the direction of the normal vector to both planes $\underset{\sim}{n} = \underset{\sim}{i} - 4\underset{\sim}{j} + 8\underset{\sim}{k}$. That is,

$$d = \left|\overrightarrow{PQ} \cdot \hat{\underset{\sim}{n}}\right| = \left|-6\underset{\sim}{i} \cdot \frac{1}{9}\left(\underset{\sim}{i} - 4\underset{\sim}{j} + 8\underset{\sim}{k}\right)\right| = \frac{2}{3}.$$

Question 16 (D)

The angle of elevation is

$$\theta = \arcsin\left(\frac{\underset{\sim}{r}(2) \cdot \underset{\sim}{k}}{|\underset{\sim}{r}(2)|}\right) = \arcsin\left(\frac{\sqrt{5}}{5}\right) = 63^\circ \quad \text{(0DP)}.$$

Question 17 (D)

Solving $\underset{\sim}{r}_1(s) = \underset{\sim}{r}_2(t)$ gives

$$\begin{cases} -1 = 2 + 3t \\ 4 + 2s = 0 \\ -s = 1 - t \end{cases} \implies s = -2 \text{ and } t = -1.$$

Hence, the lines intersect at $(-1, 0, 2)$.

Question 18 (B)

Suppose without loss of generality that the base of the tetrahedron is the triangle OBC. The height of the tetrahedron is therefore the magnitude of the scalar resolute of $\underset{\sim}{a}$ in the direction of the normal to OBC. That is,

$$V = \frac{1}{3} \cdot \frac{1}{2} |\underset{\sim}{b} \times \underset{\sim}{c}| \cdot \left| \frac{\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c})}{|\underset{\sim}{b} \times \underset{\sim}{c}|} \right| = \frac{1}{6} |\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c})|.$$

Alternatively: The volume of the tetrahedron is $\frac{1}{6}$ times the volume of the parallelepiped spanned by $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, which can be found by calculating magnitude of the scalar triple product of these vectors.

Question 19 (B)

Since X and Y are independent, we have

$$\mathrm{E}(3X - 2Y) = 3 \cdot 15 - 2 \cdot 10 = 25 \quad \text{and} \quad \mathrm{sd}(3X - 2Y) = \sqrt{3^2 \cdot 5^2 + (-2)^2 \cdot 10^2} = 25.$$

Since X and Y are normal, $3X - 2Y \sim N\left(25,\ 25^2\right)$, and therefore

$$\Pr(3X > 2Y) = \Pr(3X - 2Y > 0) = \Pr\left(Z > \frac{0 - 25}{25}\right) = \Pr(Z > -1).$$

Question 20 (D)

Option D is the definition of a type II error, so is the correct answer. **Option A** is the definition of a type I error. **Option B** and **option C** are not errors.

End of Section A

Section B

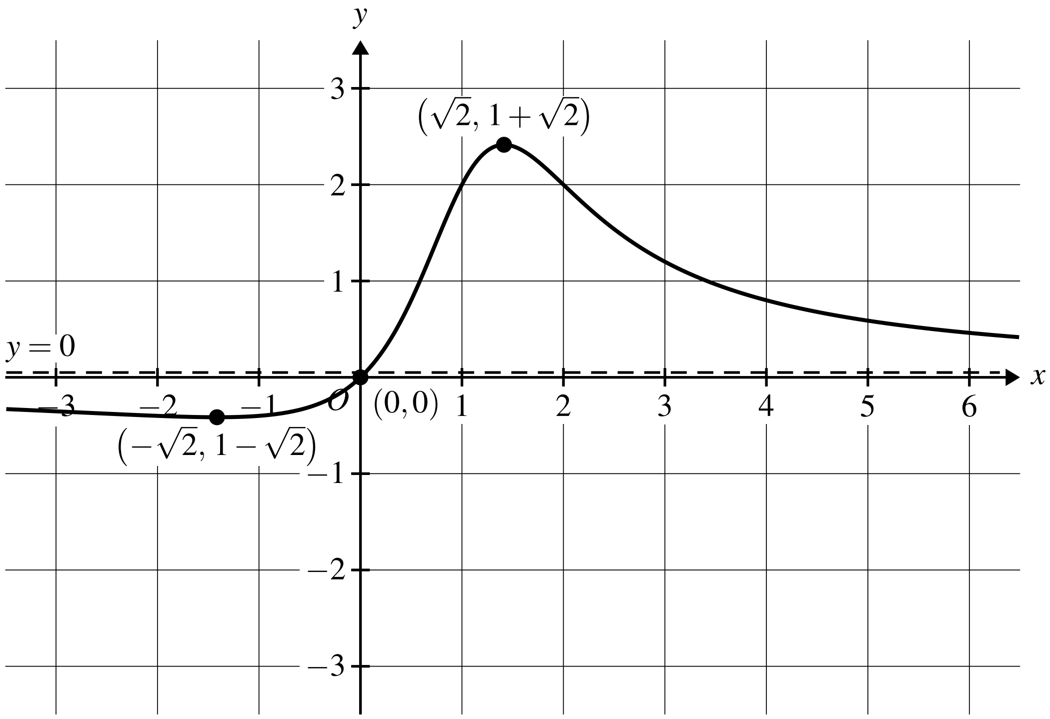
Question 1a (1 mark)

MARK 1. Labels asymptote

MARK 2. Labels stationary points and axial intercepts

MARK 3. Shows correct graph shape

Solving $f'(x) = 0$ gives $x = \pm\sqrt{2}$ and $f\left(\pm\sqrt{2}\right) = 1 \pm \sqrt{2}$. Therefore the stationary points of the graph of f are $\left(-\sqrt{2}, 1-\sqrt{2}\right)$ and $\left(\sqrt{2}, 1+\sqrt{2}\right)$.



Question 1b (2 marks)

MARK 1. Expresses volume of solid as integral

MARK 2. Provides correct answer

The volume of the solid is given by

$$V = \pi \int_0^2 \left(f(x)\right)^2 dx = 2\pi^2.$$

Question 1c (2 marks)

MARK 1. Finds $f''(x)$, or equivalent merit

MARK 2. Provides correct answer

Differentiating f twice, we get

$$f''(x) = \frac{4\left(x^3 - 6x + 4\right)}{\left(x^2 - 2x + 2\right)^3} = 0 \implies x = 2,\ -1 \pm \sqrt{3}.$$

Therefore, the points of inflection of the graph of f are $(2,2)$, $\left(-1+\sqrt{3},\ \frac{1+\sqrt{3}}{2}\right)$ and $\left(-1-\sqrt{3},\ \frac{1-\sqrt{3}}{2}\right)$.

Question 1d (1 mark)

MARK 1. Provides correct answer

Noting that g never has a removable singularity, the graph of g has exactly one vertical asymptote when the equation $x^2 + kx + 2 = 0$ has only one solution for x. This occurs when

$$\Delta = k^2 - 4 \cdot 1 \cdot 2 = 0 \iff k = \pm 2\sqrt{2}.$$

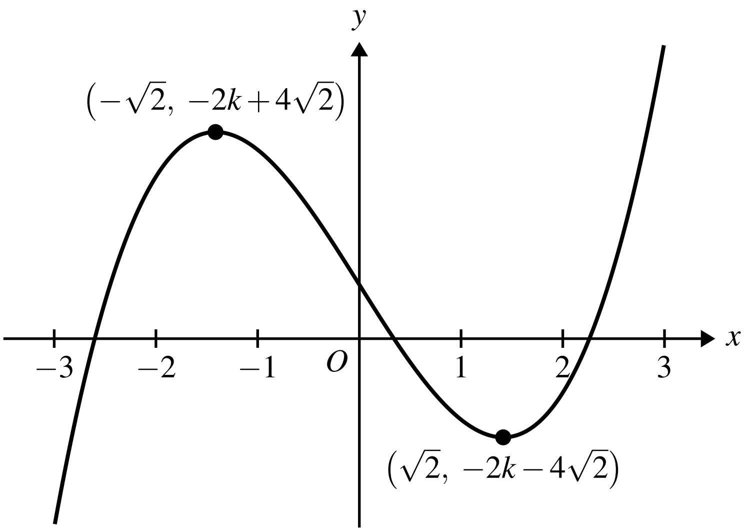
Question 1e (3 marks)

MARK 1. Relates problem to number of roots of $p(x) = x^3 - 6x - 2k$, or equivalent merit
MARK 2. Finds stationary points of graph of p, or equivalent merit
MARK 3. Provides correct answer

Consider the auxiliary function $p(x) = x^3 - 6x - 2k$. The graph of g has exactly three points of inflection if and only if p has three distinct roots. We have that

$$p'(x) = 3x^2 - 6 = 0 \implies x = \pm\sqrt{2},$$

so the stationary points of the graph of p are $\left(\sqrt{2},\ -2k - 4\sqrt{2}\right)$ and $\left(-\sqrt{2},\ -2k + 4\sqrt{2}\right)$.



Using the graph of p above, the graph of g has exactly three points of inflection if and only if

$$-2k + 4\sqrt{2} > 0 \text{ and } -2k - 4\sqrt{2} < 0 \iff -2\sqrt{2} < k < 2\sqrt{2}.$$

Question 2a (1 mark)

MARK 1. Shows sufficient and correct algebraic work to arrive at conclusion

Completing the square, we get

$$p(z) = (z-3)^3 + 3 = \left(z - 3 - \sqrt{3}i\right)\left(z - 3 + \sqrt{3}i\right).$$

Hence, the roots of $p(z)$ are $3 \pm \sqrt{3}i$.

Question 2b (1 mark)

MARK 1. Shows sufficient and correct algebraic work to arrive at conclusion

Substituting these points into the relation, we get

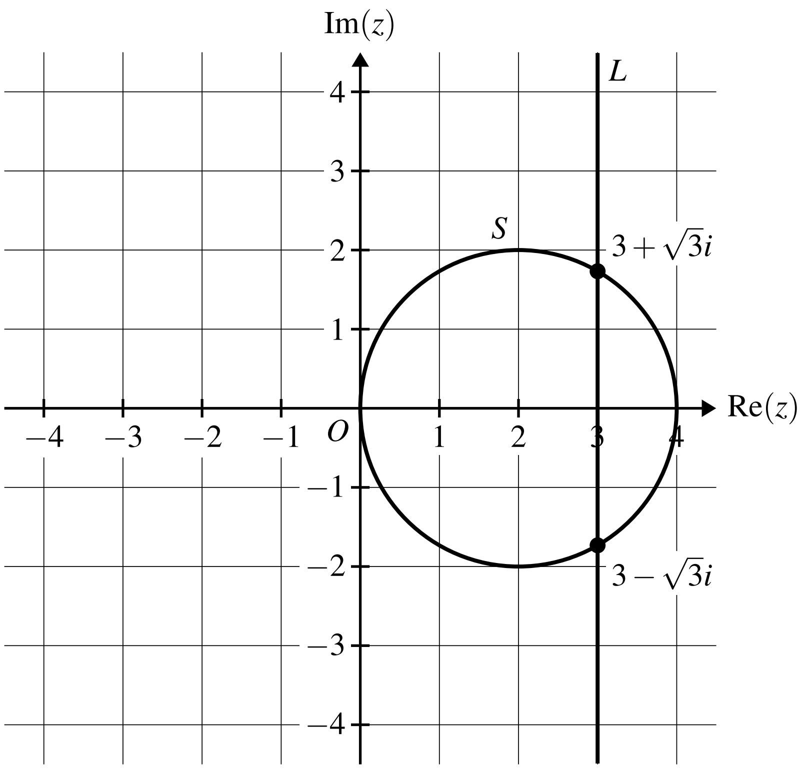
$$\left|3 \pm \sqrt{3}i - 2\right| = \left|1 \pm \sqrt{3}i\right| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2.$$

Hence, $3 \pm \sqrt{3}i \in S$.

Question 2c (2 marks)

MARK 1. Sketches S correctly
MARK 2. Labels roots of $p(z)$ correctly

Note that the diagram below contains the solution to Question 2d.i.



Question 2d.i (1 mark)

MARK 1. Sketches L correctly

The solution is shown on the diagram for the solution to Question 2c.

Question 2d.ii (2 marks)

MARK 1. Finds central angle subtending segment, or equivalent merit
MARK 2. Provides correct answer

The central angle subtending the minor segment is $\frac{2\pi}{3}$. Therefore, the area of the minor segment is

$$A = \frac{2^2}{2}\left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right) = \frac{4\pi}{3} - \sqrt{3}.$$

Question 2e (1 mark)

MARK 1. Provides correct answer

Since $b^2 - 4ac < 0$, by the quadratic formula,

$$i\bar{z} = -\frac{b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}i \implies \bar{z} = \pm\frac{\sqrt{4ac - b^2}}{2a} + \frac{b}{2a}i \implies z = \pm\frac{\sqrt{4ac - b^2}}{2a} - \frac{b}{2a}i.$$

Question 2f (2 marks)

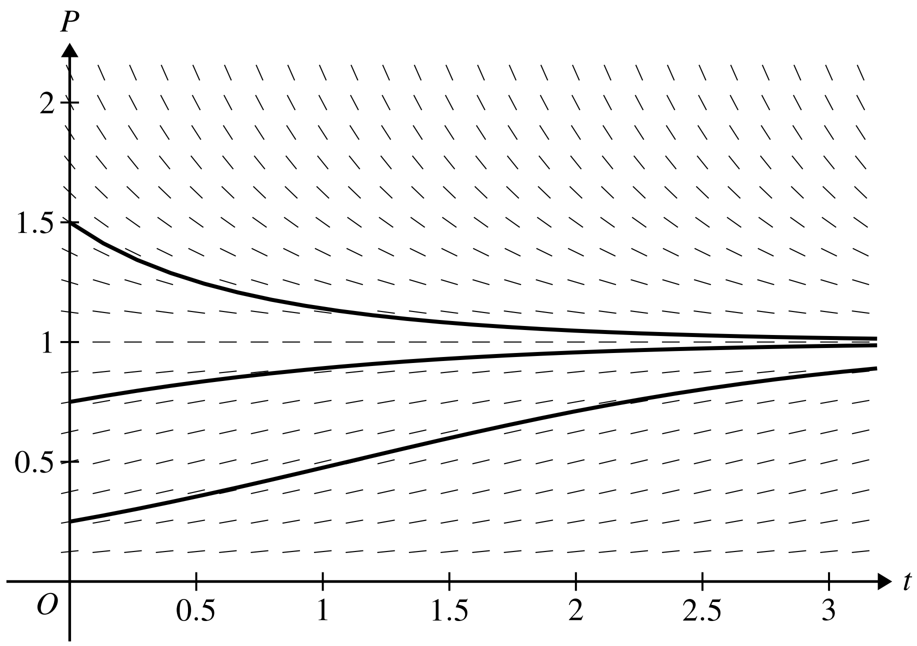
MARK 1. Reasons valid method
MARK 2. Provides correct answer

The line passing through the solutions of $q(i\bar{z}) = 0$ is given by $\text{Im}(z) = -\frac{b}{2a}i$. We also require this line to perpendicularly bisect the origin and the point represented by w. Hence,

$$w = -\frac{b}{a}i.$$

Question 3a (3 marks)

MARK 1. Sketches solution curve corresponding to $P_0 = 0.25$ correctly
MARK 2. Sketches solution curve corresponding to $P_0 = 0.75$ correctly
MARK 3. Sketches solution curve corresponding to $P_0 = 1.5$ correctly



Question 3b (1 mark)

MARK 1. Provides correct answer

The only constant solution to the differential equation is $P(t) \equiv 1$, which arises when $P_0 = 1$.

Question 3c.i (1 mark)

MARK 1. Shows sufficient and correct working to arrive at conclusion

By the chain rule,

$$\frac{d^2P}{dt^2} = \frac{dP}{dt} \cdot \frac{d}{dP}\left(P - P^2\right) = P(1-P)(1-2P).$$

Question 3c.ii (1 mark)

MARK 1. Provides correct answer

If the solution curve crosses the line $P = \frac{1}{2}$ at some $t > 0$, it will have a point of inflection. By **part a**, solution curves are monotonic. Therefore, we require that $P_0 \in \left(0, \frac{1}{2}\right)$.

Question 3d.i (3 marks)

MARK 1. Performs separation of variables, or equivalent merit
MARK 2. Establishes general solution by antidifferentiaion, or equivalent merit
MARK 3. Provides correct answer

By separation of variables and partial fraction decomposition, we have

$$t = \int_{\frac{1}{4}}^{P} \frac{1}{\theta(1-\theta)}\, d\theta = \int_{\frac{1}{4}}^{P} \left(\frac{1}{\theta} + \frac{1}{1-\theta}\right) d\theta = \Big[\log_e(|\theta|) - \log_e(|1-\theta|)\Big]_{\theta=\frac{1}{4}}^{\theta=P}.$$

Because of the initial condition, the absolute value brackets can be dropped, and so

$$t = \log_e(P) - \log_e(1-P) - \log_e\left(\frac{1}{4}\right) + \log_e\left(\frac{3}{4}\right) = \log_e\left(\frac{3P}{1-P}\right).$$

Rearranging for P, we get

$$\frac{3P}{1-P} = e^t \implies P(t) = \frac{e^t}{e^t + 3}.$$

Question 3d.ii (1 mark)

MARK 1. Provides correct answer

By **part c.i**, the point of inflection occurs where $P = \frac{1}{2}$. That is,

$$P(t) = \frac{1}{2} \implies t = \log_e(3).$$

Question 4a (2 marks)

MARK 1. Equates displacement components, or equivalent merit
MARK 2. Provides valid reasoning for conclusion

The particles collide if and only if $\underset{\sim}{r}_A(t) = \underset{\sim}{r}_B(t)$ for some $t \geq 0$. Comparing components, we get

$$3t - t^2 = 2t - 1 \iff t = \frac{1+\sqrt{5}}{2} \quad \text{and} \quad 1 - t = t \iff t = \frac{1}{2}.$$

Since $\frac{1+\sqrt{5}}{2} \neq \frac{1}{2}$, the components of the particles are never both equal. That is, $\underset{\sim}{r}_A(t) \neq \underset{\sim}{r}_B(t)$ for all $t \geq 0$, and so they do not collide.

Question 4b (2 marks)

MARK 1. Provides correct answer for particle A
MARK 2. Provides correct answer for particle B

For particle A, $t = 1 - y$ and $x = 3t - t^2$, and for particle B, $t = y$ and $x = 2t - 1$. Therefore,

$$\text{Particle A: } x = 2 - y - y^2 \quad \text{and} \quad \text{Particle B: } x = 2y - 1.$$

Question 4c (1 mark)

MARK 1. Provides correct answer

Due to the restriction $t \geq 0$ not that the paths of the particles cross where $y \geq 0$. Therefore

$$2 - y - y^2 = 2y - 1 \implies y = \frac{\sqrt{21} - 3}{2} \implies x = \sqrt{21} - 4.$$

Hence the paths of the particles cross at $(0.58, 0.79)$ (2DP).

Question 4d (3 marks)

MARK 1. Expresses distance in terms of t, or equivalent
MARK 2. Provides correct answer for time when minimum distance occurs
MARK 3. Provides correct answer for minimum distance

The distance between the particles after t seconds is given by

$$d(t) = \left|\underset{\sim}{r}_A(t) - \underset{\sim}{r}_B(t)\right| = \sqrt{t^4 - 2t^3 + 3t^2 - 2t + 2}.$$

Then, the minimum distance between the particles occurs when

$$d'(t) = \frac{2t^3 - 3t^2 + 3t - 1}{\sqrt{t^4 - 2t^3 + 3t^2 - 2t + 2}} = 0 \implies t = \frac{1}{2} \text{ s},$$

and this minimum distance is

$$d_{\min} = d\left(\frac{1}{2}\right) = \frac{5}{4} \text{ m}.$$

Question 4e (2 marks)

MARK 1. Establishes equation in terms of t

MARK 2. Provides correct answer

The speeds of the particles are equal when

$$\left|\underset{\sim}{r}'_{A}(t)\right| = \left|\underset{\sim}{r}'_{B}(t)\right| \implies \sqrt{4t^2 - 12t + 10} = \sqrt{5} \implies t = \frac{1}{2}, \frac{5}{2}.$$

Question 5a (2 marks)

MARK 1. Finds a normal vector to plane, or equivalent merit

MARK 2. Shows sufficient and correct working to arrive at conclusion

Two vectors parallel to the plane are $\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$ and $-\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$, so a normal vector to the plane P is

$$\underset{\sim}{n} = \left(\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}\right) \times \left(-\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}\right) = \begin{vmatrix} -3 & 2 \\ -3 & -2 \end{vmatrix}\underset{\sim}{i} - \begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix}\underset{\sim}{j} + \begin{vmatrix} 1 & -3 \\ -1 & -3 \end{vmatrix}\underset{\sim}{k} = 12\underset{\sim}{i} - 6\underset{\sim}{k}.$$

The Cartesian equation of P is therefore given by $12x - 6z = c$, for some $c \in \mathbb{R}$. Substituting in the point $(0,2,2)$ gives $c = -12$, so

$$12x - 6x = -12 \implies -2x + z = 2.$$

Question 5b (1 mark)

MARK 1. Provides correct answer with justification

The plane P is perpendicular to the x-z plane since $\underset{\sim}{n} \cdot \underset{\sim}{j} = 0$.

Question 5c (2 marks)

MARK 1. Finds parameter value for intersection point, or equivalent merit

MARK 2. Provides correct answer

Substituting $\underset{\sim}{r}(t)$ into the plane equation gives

$$\underset{\sim}{r}(t) \cdot \left(-2\underset{\sim}{i} + \underset{\sim}{k}\right) = 2 \implies t = 1.$$

Therefore the line intersects the plane at $\underset{\sim}{r}(1) = (-1, 3, 0)$.

Question 5d (3 marks)

MARK 1. Relates orthogonal components to vector resolute in direction of $\underset{\sim}{n}$, or equivalent merit
MARK 2. Provides correct answer for parallel component
MARK 3. Provides correct answer for perpendicular component

Let $\underset{\sim}{v} = -2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$. The orthogonal components of $\underset{\sim}{v}$ with respect to the plane P are the vector resolutes of $\underset{\sim}{v}$ perpendicular and parallel to the normal vector $\underset{\sim}{n}$ of the plane. Therefore,

$$\underset{\sim}{v} = \left(\underset{\sim}{v} - (\underset{\sim}{v} \cdot \hat{\underset{\sim}{n}})\,\hat{\underset{\sim}{n}}\right) + (\underset{\sim}{v} \cdot \hat{\underset{\sim}{n}})\,\hat{\underset{\sim}{n}} = \underset{\sim}{j} + \left(-2\underset{\sim}{i} + \underset{\sim}{k}\right),$$

where $\underset{\sim}{j}$ is the component of $\underset{\sim}{v}$ parallel to P and $-2\underset{\sim}{i} + \underset{\sim}{k}$ is the component of $\underset{\sim}{v}$ perpendicular to P.

Alternatively: from ***part b****, the vector* $\underset{\sim}{j}$ *is parallel to P so the perpendicular component must be* $-2\underset{\sim}{i} + \underset{\sim}{k}$.

Question 5e (2 marks)

MARK 1. Finds either direction vector of line or point on line
MARK 2. Provides correct answer

A direction vector of L' is the reflection of a direction vector of L in the plane P. Taking the negative of the perpendicular component in **part d.** gives

$$\underset{\sim}{u} = \underset{\sim}{j} - \left(-2\underset{\sim}{i} + \underset{\sim}{k}\right) = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$$

From **part c**, the point $(1, 3, 0)$ must also lie on L', so a vector equation for L' is

$$\ell(s) = -\underset{\sim}{i} + 3\underset{\sim}{j} + s\left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right), \quad s \in \mathbb{R}.$$

Note: there are several possible correct answers, and taking the negative of the parallel component instead for the direction vector also works due to symmetry.

Question 6a (1 mark)

MARK 1. Provides correct answer

Let $H_1, H_2 \sim N\left(3.2, 0.45^2\right)$ be the heights of plants in centimetres. Then,

$$\Pr(\overline{H} > 3.1) = 0.6233 \quad \text{(4DP)}.$$

Question 6b (3 marks)

MARK 1. Finds distribution of height difference, or equivalent merit
MARK 2. Applies definition of conditional probability
MARK 3. Provides correct answer

Let $D = H_1 - H_2$ be the (signed) difference in the heights of the plants. Since H_1 and H_2 are independent, we have $D \sim N\left(0, (0.45\sqrt{2})^2\right)$. Therefore, by symmetry

$$\Pr\left(|D| < 0.2 \mid |D| > 0.1\right) = \frac{2\Pr(0.1 < D < 0.2)}{2\Pr(D > 0.1)} = \frac{0.1218222...}{0.8751386...} = 0.1392 \quad \text{(4DP)}.$$

Question 6c (1 mark)

MARK 1. Provides correct answer

Suitable hypotheses for this test are $H_0 : \mu = 3.2$ m and $H_1 : \mu \neq 3.2$ m.

Question 6d (1 mark)

MARK 1. Provides correct answer

Let $\overline{X}$ be the random variable representing the mean height of the plants in the sample. The p value for the test is given by

$$p = 2\Pr\left(\overline{X} > 3.3 \mid \mu = 3.2\right) = 0.0852 \quad \text{(4DP)}.$$

Question 6e (1 mark)

MARK 1. Provides correct answer with justification

As $p > 0.05$, this sample does **not** provide evidence at the 5% level of significance that the new fertiliser changes the mean height of the plants.

Question 6f (2 marks)

MARK 1. Reasons valid method
MARK 2. Provides correct answers

The acceptance region for this test is the interval $[c, d]$ centred on $\mu = 3.2$ such that

$$\Pr\left(c \leq \overline{X} \leq d \mid \mu = 3.2\right) = 0.95 \implies \begin{cases} \Pr\left(\overline{X} < c \mid \mu = 3.2\right) = 0.025 \\ \Pr\left(\overline{X} < d \mid \mu = 3.2\right) = 0.975 \end{cases}.$$

Using the inverse normal cumulative distribution function, the critical sample means are therefore

$$c = 3.0861 \text{ m} \ \text{ and } \ d = 3.3139 \text{ m} \quad \text{(4DP)}.$$

End of Solutions Book

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